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P34

Fuzzy Logic: A Tutorial

Ray Wicks

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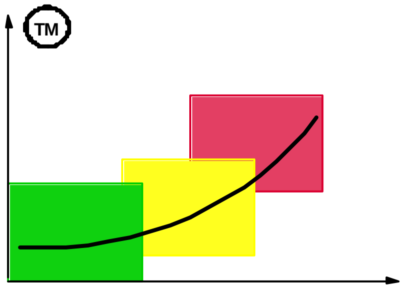

Fuzzy Logic

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Abstract

Fuzzy Logic: A tutorial
In a course in switching theory or traditional symbolic logic, one studies a form of logic which has existed from the early Greeks, notably Aristotle. This session reviews the principles of this crisp symbolic logic (negation, and, or, if-then, etc.) and then proceeds to introduce Fuzzy logic and Fuzzy sets. This new logic has interesting ramifications in fuzzy thinking and neural networks. An example using fuzzy rules in a control system will be introduced. You don't need a logic background for this session.

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Background

Everyone in their own mind thinks of themselves as logical, clear thinking and free from bias.

In the middle ages, logic was taught as a method of arriving at correct answers from accepted assumptions. Metaphysics provided the unassailable assumptions. The approach was impeccable.

However, semantics of actual language is not binary (black and white) but fuzzy (shades of grey). The introduction of fuzziness into logic creates havoc among logicians but brings logic closer to every day usage.

Crisp Definitions

Not P

P	~P
0	1
1	0

P or Q

P	Q	PvQ
0	0	0
0	1	1
1	0	1
1	1	1

P and Q

P	Q	P?Q
0	0	0
0	1	0
1	0	0
1	1	1

If P then Q

P	Q	P? Q
0	0	1
0	1	1
1	0	0
1	1	1

Game Rules

P	Q	P * Q
T	T	T
T	⊥	⊥
⊥	T	⊥
⊥	⊥	⊥

Crisp Definitions

P is equivalent to Q

P	Q	P≡Q
0	0	1
0	1	0
1	0	0
1	1	1

P? Q.?.Q? P

P Excl or Q

P	Q	P ⊻ Q
0	0	0
0	1	1
1	0	1
1	1	0

P⊻Q = P?~Q.v.~P?Q

Or

P⊻Q = ~(P≡ Q)

Reduction a la Russell

Using "|" = "is incompatible with" or "not both"

P Stroke Q

P	Q	P | Q
0	0	1
0	1	1
1	0	1
1	1	0

~P = P|P

PvQ = ~P|~Q

P?Q = ~(P|Q)

P? Q = P|~Q

Obscurities

If p or q and If q Then r, Then p or r.

((pvq) ? (q? r)) ? (pvr)

pvq.?.q? r:? :pvr

CKApqCqrApr

Truth Analysis – A Game

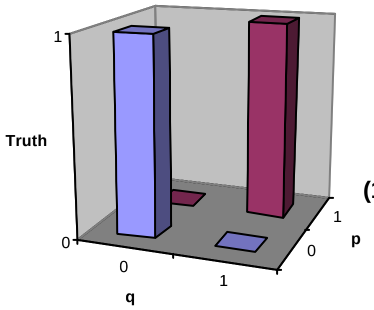
v	0	1
0	0	1
1	1	1

?	0	1
0	0	0
1	0	1

?	0	1
0	1	1
1	0	1

- **PVQ**
  - **1vQ replace with 1**
  - **0vQ replace with Q**
- **P?Q**
  - **1?Q replace with Q**
  - **0?Q replace with 0**
- **P? Q**
  - **P? 0 replace with ~P**
  - **P ? 1 replace with 1**
  - **1 ? Q replace with Q**
  - **0 ? Q replace with 1**

Truth Analysis (Crisp)



If p or q Then p and q

pvq :? : p? q

p

1 **0**

(1vq) .? . 1 ? q **(0vq) .? . 0 ? q**

1 .? . q **q .? . 0**

q **~q**

Logical Deduction

Modus Ponens

⊢ **P ? Q** ⊢ **P** ∴ **Q**	**or**	**P ? Q** **P** — **Q**

If I eat candy, my blood sugar will rise

I ate candy

My blood sugar is elevated

Logical Deduction
Modus Tollens

P ? Q

~Q

~P

If I eat candy, my blood sugar will rise

My blood sugar is not elevated

I did not eat candy

Logical Deduction
Modus Tollendo Ponens

P v Q

~P

Q

I took a walk or my blood sugar is elevated

My blood sugar is not elevated

I took a walk

Note: In symbolic logic, the sentences (p,q) need not be semantically linked.

Logical Fallacies

Fallacy of Affirming the Consequent

P ? Q

Q

P

If I eat candy, my blood sugar will rise

My blood sugar is elevated

I must have eaten candy

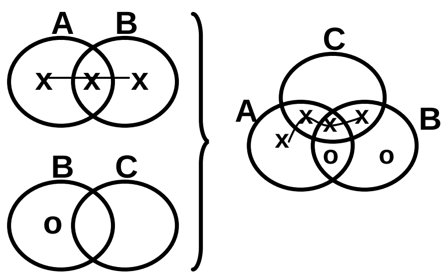
Quantitative & Visual Logic

$\Sigma \mathbf{x.A_x \ v \ B_x}$

For some x, x has property A or x has Property B

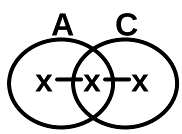
$\Pi \mathbf{x.B_x} \subset \mathbf{C_x}$

For all x, if x has property B thenr x has Property C



$\Sigma \mathbf{x.A_x \ v \ C_x}$

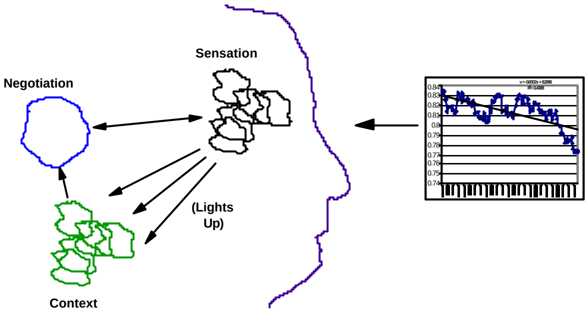
For some x, x has property A or x has Property C



Semantic Evaluation of a CS

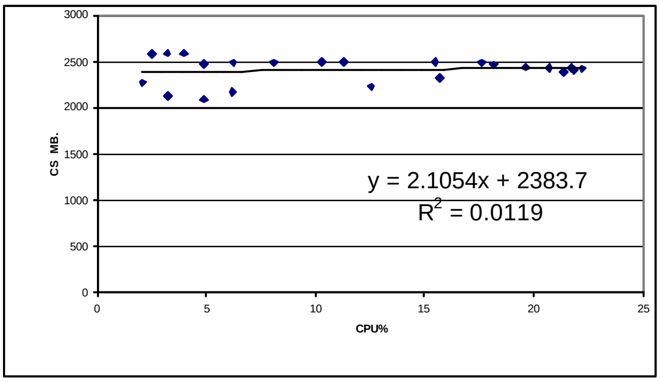
- **Consistent - Nothing Logically absurd or self contradictory in meaning shall be a theorem, or that there shall not be two theorems of which one is the negation of the other. (p v ~p is not a theorem).**
- **Complete – Roughly, the system shall have all possible theorems not in conflict with the interpretation. (You can prove what you want to prove.)**
- **Simple**
- **Pragmatic**
- **Verifiable**

Philosophical Remark



In reaching a conclusion, we negotiate between the potential perceptual structures and the potential conceptual structures and memory events.

Our Perception is Fuzzy.
Is Our Thinking Fuzzy?



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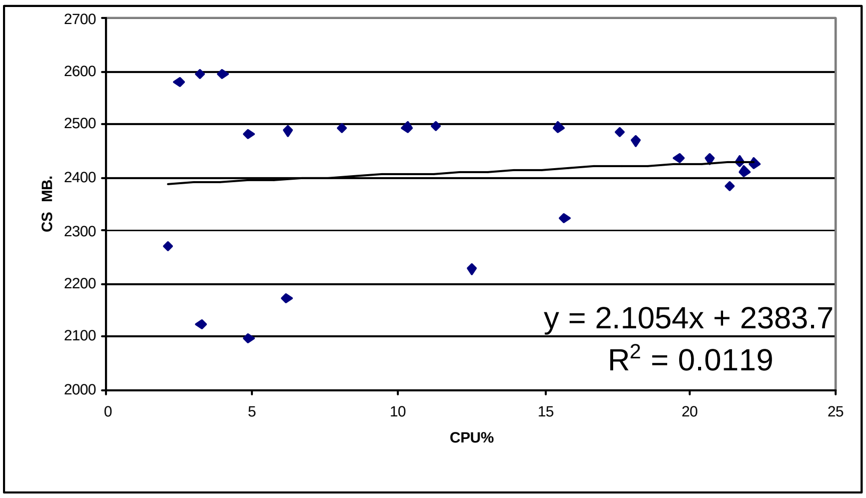


Paradox: *P v ~P* ?

The Liar: **'A man says he is lying. Is what he says true or false?'**

The heap: **If you have a heap of sand and remove one grain at a time, when does it cease to be a heap?**

***Parking*: If you park in a parking lot and do not park exactly in the space between the lines, are you parked in the space or not?**

In Ordinary Language

C: "How's the wine?"

F: "Pretty good."

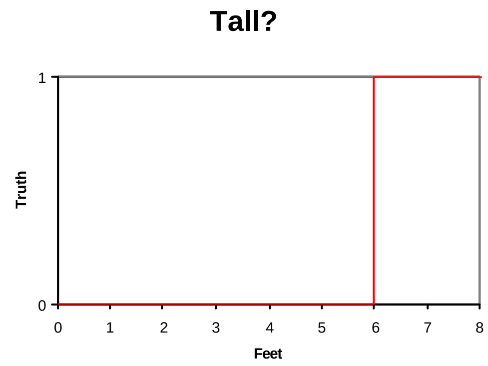
C: "How's the cheese?"

F: "Not bad."

C: "Isn't anything ever just good or bad with you?"

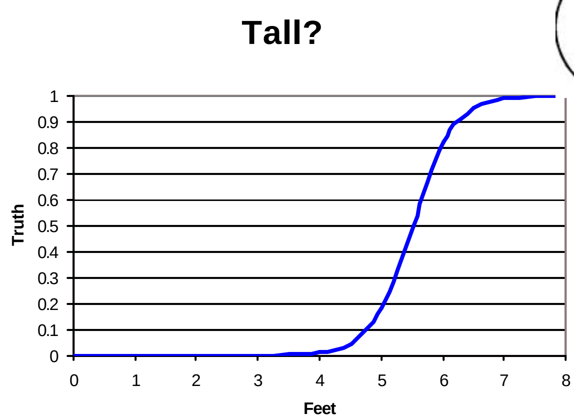
F: "Sometimes."

Can Truth Be a Matter of Degree?



If the rule is Tall > 6', what about 5.99'? 5.95'?... One gets the feeling that the rule is not necessarily crisp since even the measurement is itself fuzzy.

Can Truth Be a Matter of Degree?



To represent the Fuzzy rule, a non binary function is available which does not yield either 0 or 1 but a value on the interval [0,1]. Can you stand something being not necessarily true or false?

Hedge Words

"Joe Montana ***always*** came through in the clutch."

Hedge Word	Avg	Min	Max	CV
Always	98.08	95	100	0.022
Very often	87.15	70	95	0.07
Rather often	78.08	65	85	0.09
Usually	76.08	60	85	0.093
Often	73.38	50	90	0.166
Frequently	72.54	50	87	0.139
Generally	71.92	60	85	0.104
About as often as not	46.54	5	50	0.268
Sometimes	40.77	20	60	0.267
Occasionally	36.15	15	65	0.442
Now and then	31.54	5	45	0.385
Once in a while	27.08	5	60	0.587
Not often	22.67	10	35	0.352
Usually not	19.69	6	40	0.538
Seldom	16.92	8	30	0.45
Rarely	13.23	3	30	0.541
Almost never	12.69	2	85	1.736
Hardly ever	11.23	1	30	0.65
Very seldom	10.92	5	20	0.455
Never	1.231	0	5	1.526

Fuzz is Not Probability

In **probability** you deal with frequencies of events. When you throw a pair of dice, the probability of getting a 2 is 1 in 36. You will get one of the values.

In **fuzzy** you deal with degrees. When you fill a glass ¾ full, is it full or not? Yes to 0.75 degree.

Fuzzy Logic

P,Q ε {0,1}

P	~P
0	1
1	0

P	Q	PvQ
0	0	0
0	1	1
1	0	1
1	1	1

P,Q ε [0,1]

Not P = 1-P

~0.7 = 0.3

PvQ= Max(P,Q)

0.3 V 0.75 = 0.75

Fuzzy Logic

P,Q ♏ {0,1}

P	Q	P?Q
0	0	0
0	1	0
1	0	0
1	1	1

P	Q	P? Q
0	0	1
0	1	1
1	0	0
1	1	1

P,Q ♏ [0,1]

P?Q = Min(P,Q)

0.7?0.3 = 0.3

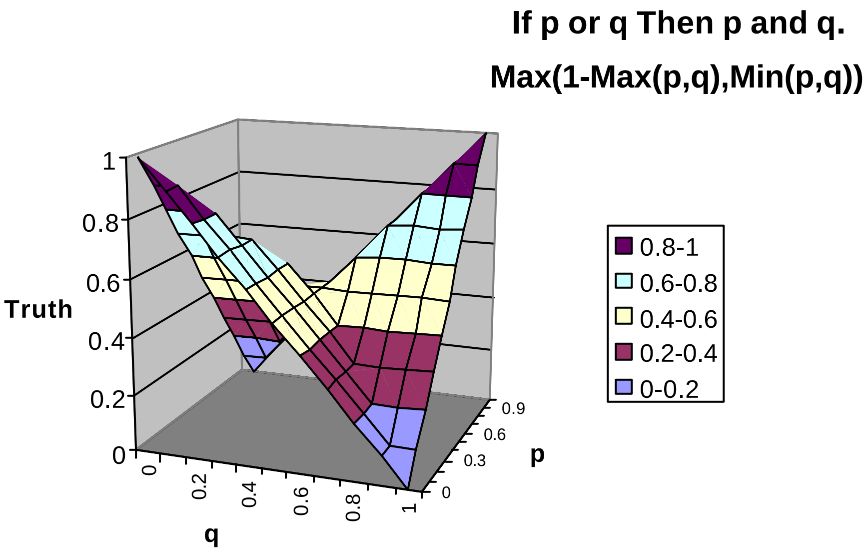
P? Q= ~(P?~Q)

= ~P v Q

= Max(1-P,Q)

0.75 ?   0.5 = 0.5

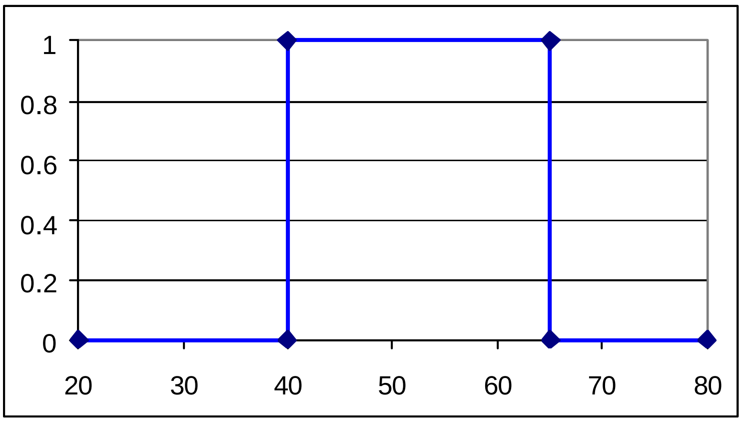
Truth Analysis (Fuzzy)



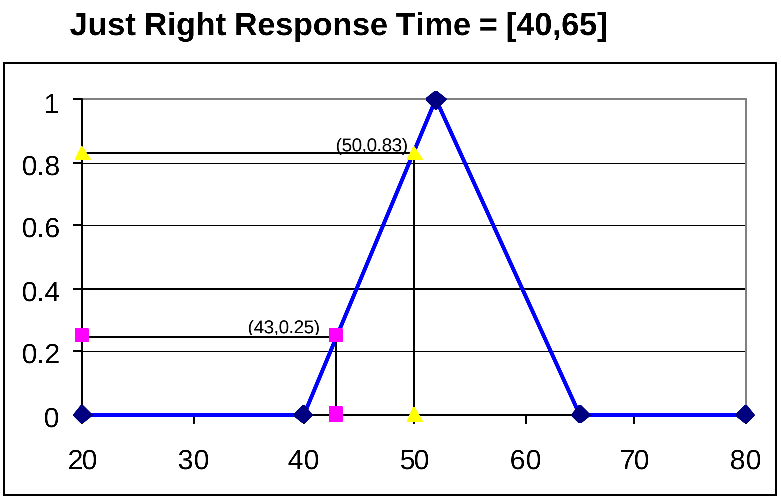
Note: the corners match the crisp version.

Crisp Set

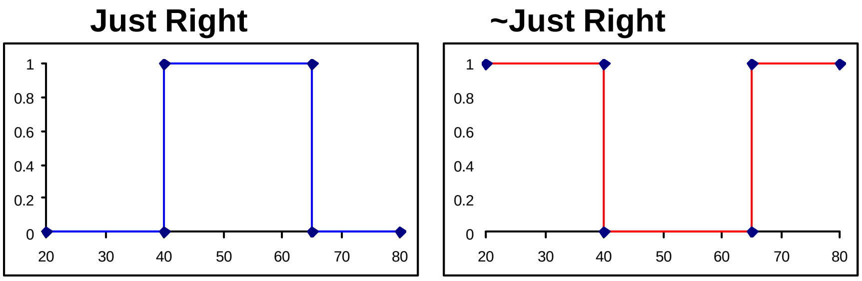
Just Right Response Time = [40,65]



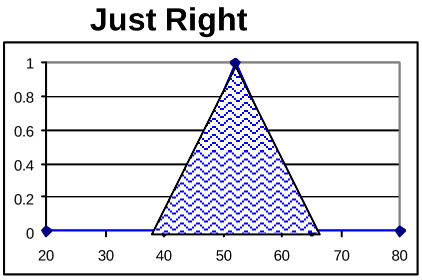
Fuzzy Set

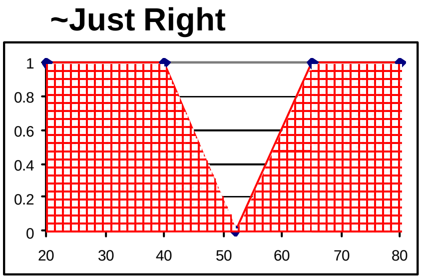


Crisp ~Just Right

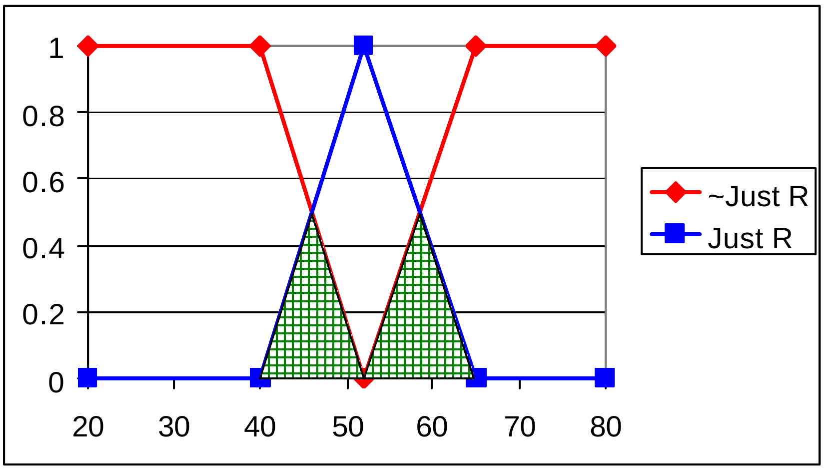


Fuzzy ~Just Right





Fuzzy Just Right and ~JR



Fuzzy Rules For Response Time

Rule 1: If too short, lower priority a lot.

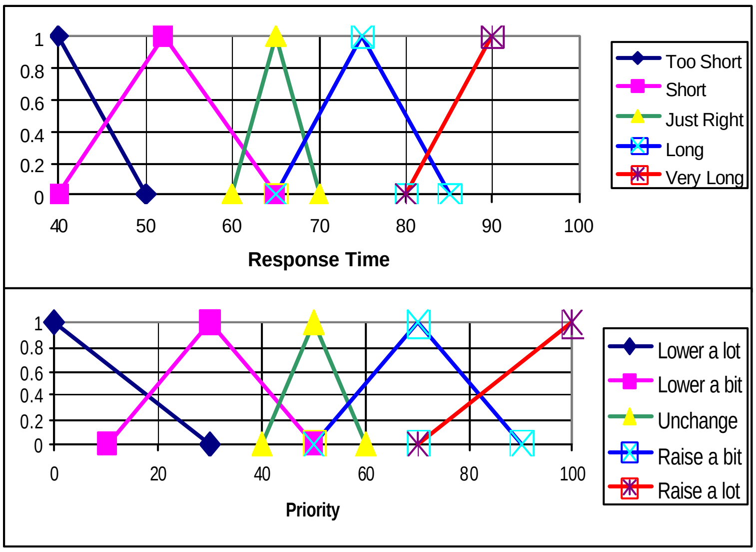
Rule 2: If short, then lower priority a bit.

Rule 3: If just right, then leave priority unchanged.

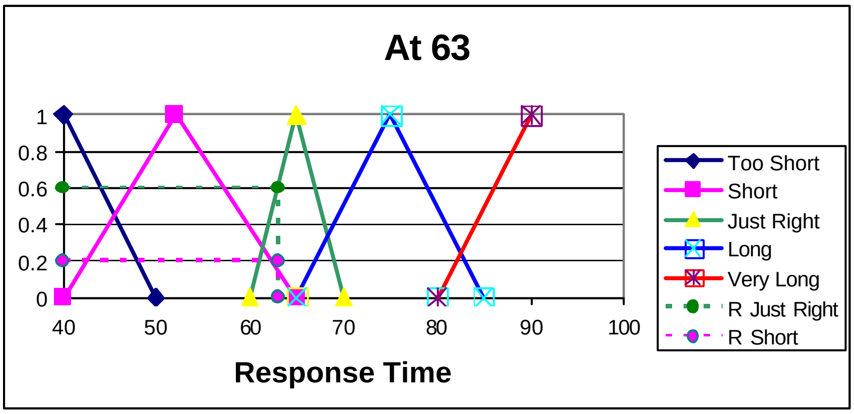
Rule 4: If long, then raise priority a bit.

Rule 5: If very long, then raise priority a lot.

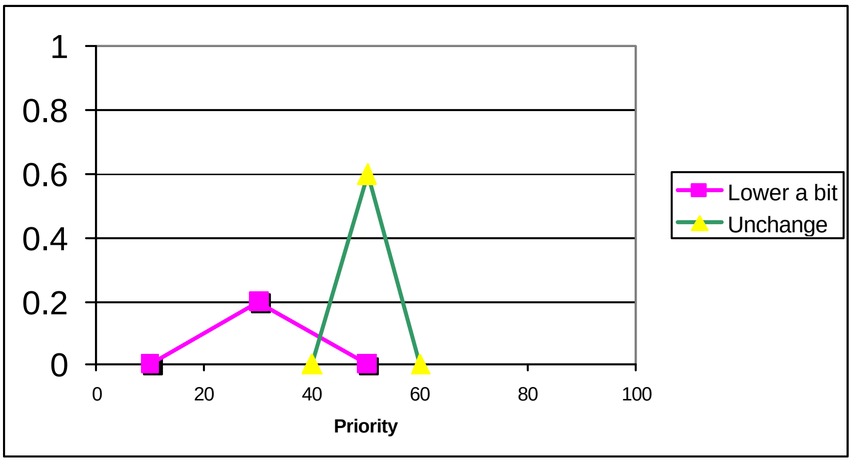
Graph of Fuzzy Rules



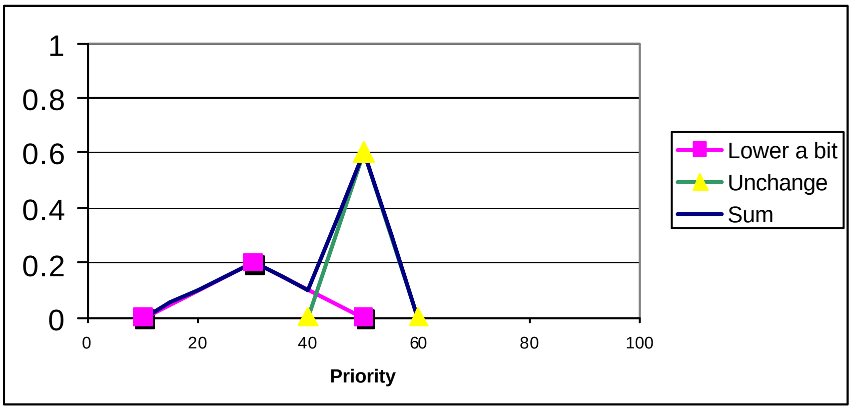
Hedge for Response 63



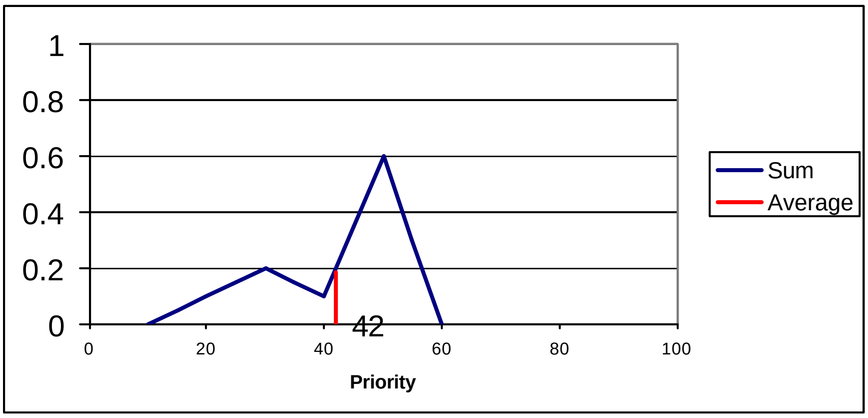
Response Rules Fire



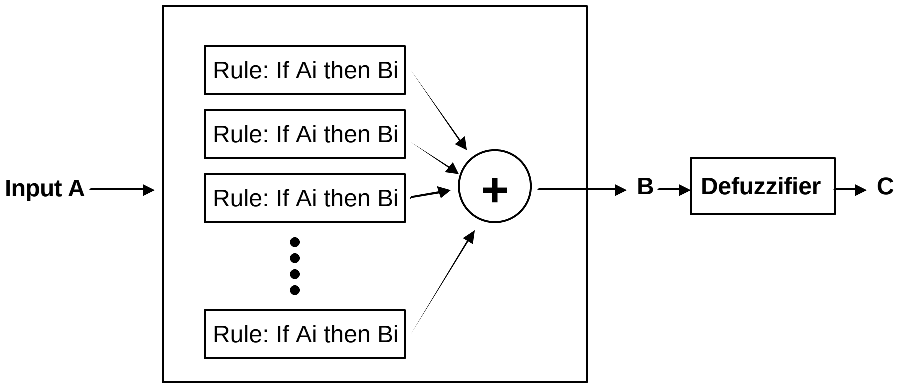
Add Rule Output



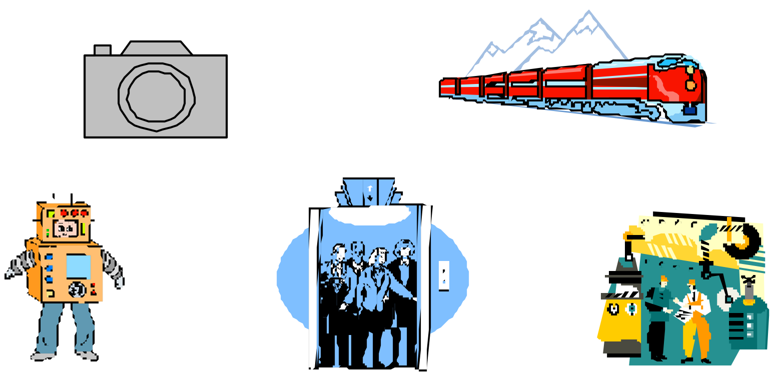
DeFuzzify the Result



Process



Applications



SYS1	SYS2
155.3	148.1
175.3	129.8
144.7	127.5
132.4	134.3
150.7	143.1
121.2	117.3
124.1	124.9
123.6	117.5
156.3	154.1
129.9	133.3
136.6	134.8
126.6	128.7
154.4	144.4
128.6	117.7
127.5	121.4
119.8	118.5
153.8	143.3
128	118.1
132.2	127.5
133.7	120.4
147.3	137.5
140.3	114.2
121.4	114.4
117.4	108.9
158	144
127.6	118.9
129.5	125.5
126.3	116.8
168.8	137.8
122.8	110.1
123.1	110.7
117.9	113.8
149.6	139.8
129.9	115.1
129.3	118.5
125.3	118.6
141.1	133.7
119.4	107.5
117	107.2
112.9	102.5

Fuzzy Statistics
T-Test @ 0.05

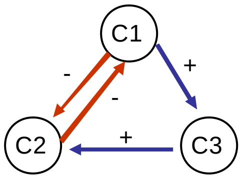
t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	134.49	125.005
Variance	231.5271	165.2302
Observations	40	40
Hypothesized Mean Difference	0	
df	76	
t Stat	3.011653	
P(T<=t) one-tail	0.001763	
t Critical one-tail	1.665151	
P(T<=t) two-tail	0.003527	
t Critical two-tail	1.991673	

What about 0.06? 0.08? 0.025?

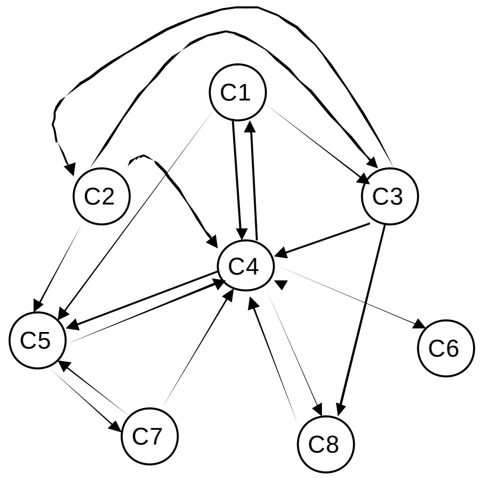
Fuzzy Cognitive Maps

“Fuzzy cognitive maps (CFMs) are fussy signed directed graphs with feedback. The directed edge e_{ij} from causal concept C_i to concept C_j measures how much C_i causes C_j. The time-varying concept $C_i(t)$ measures the nonnegative occurrence of some fuzzy event.” [Kosko 2]



Fuzzy Cognitive Maps

C1=Football
C2= Alcohol Consumption
C3=Raised Testosterone
C4=Competitive Nature
C5=Respect for Women
C6=Long Term Relationship
C7=Practical Jokes
C8=Video Games



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CFM Computations

C1=Football
C2= Alcohol Consumption
C3=Raised Testosterone
C4=Competitive Nature
C5=Respect for Women
C6=Long Term Relationship
C7=Practical Jokes
C8=Video Games

Always = 1
Usually= 0.8
Frequently = 0.6
Sometimes = 0.4
Rarely = 0.2
Never = 0

	C1	C2	C3	C4	C5	C6	C7	C8	
C1	1	0	1	1	-0.4	0	0	0	
C2	0	0	0.8	0.6	-1	0	0	0	
C3	0	0.6	0	1	0	0.6	0	0	
C4	0.2	0	0.8	0	-0.6	1	0	0.8	
C5	0	0	0	-0.6	0	0	0.8	0	
C6	0	0	0	0.2	0	0	0	0	
C7	0	-0.2	-0.4	-0.8	0.8	0	0	0	
C8	0	0	0	1	0	0	0	0	
S0	1	0	0	0	0	0	0	0	
S1	1	0	1	1	-0.4	0	0	0	
S1*	1	0	1	1	0	0	0	0	DeFuzzify
S2	1.2	0.6	1.8	2	-1	1.6	0	0.8	
S2*	1	1	1	1	0	1	0	1	DeFuzzify
S3	1.2	0.6	2.6	3.8	-2	1.6	0	0.8	
S3*	1	1	1	1	0	1	0	1	DeFuzzify

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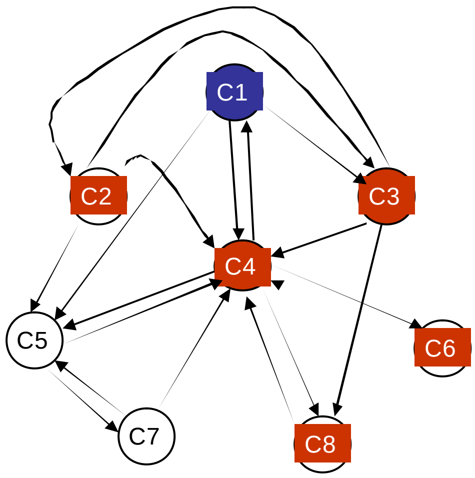

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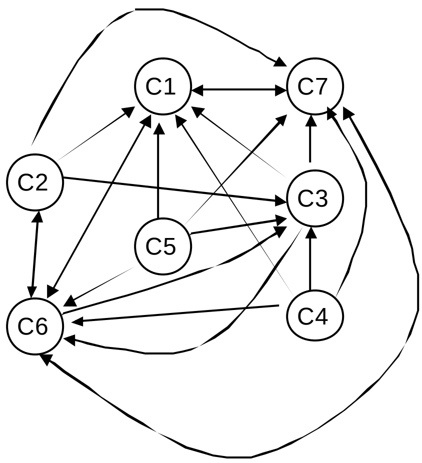
CFM

C1=Weight Loss
C2= Healthy Diet
C3=Not Smoking
C4=Avoiding Alcohol
C5=Exercise
C6=Decreased Stress
C7=Increased Energy



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CFM Computations

C1=Weight Loss

C2= Healthy Diet

C3=Not Smoking

C4=Avoiding Alcohol

C5=Exercise

C6=Decreased Stress

C7=Increased Energy

	C1	C2	C3	C4	C5	C6	C7
C1	0	0	0	0	0	0.3	0.7
C2	0.3	0	0.1	0.1	0	0.3	0.2
C3	-0.2	0	0	0	0	-0.3	0.9
C4	0.6	0	0.1	0	0	-0.2	0.5
C5	0.5	0	0.1	0	0	0.2	0.2
C6	-0.2	0.1	0.6	0.1	0	0	0.5
C7	0.6	0	0	0	0	0.4	0
S1	1	0	0	0	0	0	0
S2	0	0	0	0	0	0.3	0.7
S2*	1	0	0	0	0	0	1
S3	0.6	0	0	0	0	0.7	0.7
S3*	1	0	0	0	0	1	1
S4	0.4	0.1	0.6	0.1	0	0.7	1.2
S4*	1	0	1	0	0	1	1
S5	0.2	0.1	0.6	0.1	0	0.4	2.1
S5*	1	0	1	0	0	0	1
S6	0.4	0	0	0	0	0.4	1.6
S6*	1	0	0	0	0	0	1

CFM

C1=Weight Loss

C2= Healthy Diet

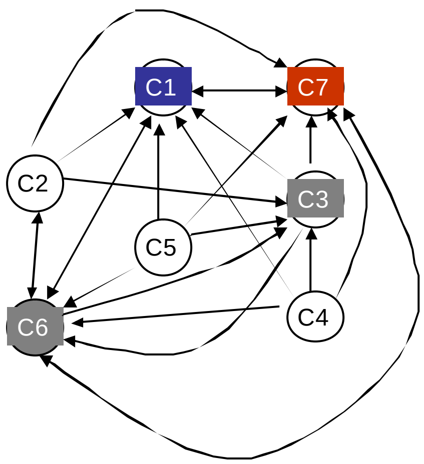
C3=Not Smoking

C4=Avoiding Alcohol

C5=Exercise

C6=Decreased Stress

C7=Increased Energy



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